1 Description of Files

1.1 Main Simulation Files

LP_main: This file includes the codes to replicate Table 2, Fig 2-5 in the paper.

LP_CompareWithMogstadEtAl: this file includes the codes to replicate Fig 1 in the paper.

LP_MTE_diffK: this file includes the codes for the left subfigure in Fig 6.

LP_CompareWithManski: this file includes the codes for the right subfigure in Fig 6.

LP_Continuous: this file includes the codes for Fig 9.

LP_CompareWithM_DiffWidth: this file includes the codes for Fig 10.

LP_CommonW: this file includes the codes for Fig 11.

LP_DiffK_Misspecification: this file includes the codes for Fig 11 - Fig 12.

1.2 Main Function Files

bern: this function generates basis functions for Bernstein polynomials.

bern3: this function creates multivariate Bernstein polynomial basis functions with three components, primarily used in simulations involving continuous Y.

ChooseRS: this function determines the direction for Assumption U without W, and generates corresponding target and restriction coefficient matrices.

ChooseRSW_new: this function determines the direction for Assumption U with W, and generates corresponding target and restriction coefficient matrices.

ChooseU0: this function determines the direction for Assumption U0 with W, and generates corresponding coefficient matrices for additional inequality constraints.

ChooseRSW: this function determines the direction for Assumption U* with W, and generates corresponding target and restriction coefficient matrices.

2 Details of Each File

2.1 LP_main

This file contains the codes to replicate Table 2 and Fig 2-5 from the paper. It creates partially identified bounds on ATE and LATEs under various assumptions.

- The DGP used here is consistent with that introduces in Section 8.1 of the paper.
- We applied Bernstein polynomials of order up to $K = 50$ for sieve approximation. This choice is related to the sensitivity of MTE bound to K, as discussed under Fig 6.
- When Assumption U0, U, or U* is assumed, the direction is determined via a data-driven approach using the functions *ChooseU0*, *ChooseRSW_new* (or *ChooseRS*) and *ChooseRSW* .
  - Results under Assumption U with W are not presented in the graph as they are identical to those under Assumption U0. However, code and results for Assumption U are included in the file for comparison purpose.
  - We allowed the directions to vary across covariate X
- A represents the target coefficient matrix for ATE.
- $LATE$ is the coefficient matrix for LATEs, where
  - $LATE(\cdot, \cdot, 1)$ represents the coefficients in front of θ for the always taker LATE.
  - $LATE(\cdot, \cdot, 2)$ represents the coefficients for the complier LATE.
  - $LATE(\cdot, \cdot, 3)$ represents the coefficients for the never taker LATE.
- B is the coefficient matrix in the data restriction as shown in (LP3).
- *one* is the coefficient matrix for probability restriction such that the sum of probabilities of different types equals to 1.
- M, C represent coefficient matrices for the shape restrictions (Assumption M and Assumption C respectively);
- G_M is the coefficient matrix for additional inequality restriction under Assumption U0.

2.2 LP_CompareWithMogstadEtAl

This file compares the current approach with Mogstad et al. (2018) and generates Fig 1 in the paper.

- For a fair comparison, we implement a DGP without W. The DGP includes binary D and discrete Z, Y. We vary support for Z and Y to create different scenarios. Specifically, Y takes values in $\{0,1\}$, $\{0,0.5,1\}$, $\{0,0.25,0.5,0.75,1\}$, $\{0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9\}$ and Z takes values in $\{0,1\}$, $\{0,0.5,1\}$, $\{0,0.25,0.5,0.75,1\}$.

- For Mogstad et al. (2018), we use the IV-like estimand for the sharp bound, as described in Section 2.5 in their paper. Specifically, we applied the following set:

$$\mathcal{S} = \{1(d=i, z=j), i \in \{1,2\}, j \in \mathcal{Z}\}.$$

2.3 LP_MTE_diffK

This file presents the convergence of partial identification bounds as the polynomial order K increases.

- We consider a DGP without W, and compare the bounds on MTE under Assumption U and shape restrictions. The bounds under other assumptions present a similar pattern.

- Names for coefficient matrices are consistent with the ones in *LP_main*.

2.4 LP_CompareWithManski

This file generates bounds on ATE for various K and compares them with the closed-form bound by Manski (1990).

- To ensure a fair comparison, we create the MTR functions following a inverse normal distribution function, rather than directly using Bernstein polynomials.

- We compare the bounds with K varying from 10-50

- Manski's bounds are calculated as:

$$
\begin{aligned}
LB = &\sup_{z\in\mathcal{Z}} \left\{ P(z)E\left[Y_1|D=1,Z=z\right] + \left(1-P(z)\right) y^l_{z,1} \right\} \\
&- \sup_{z\in\mathcal{Z}} \left\{ \left(1-P(z)\right) E\left[Y_0|D=0,Z=z\right] + P(z) y^u_{z,0} \right\} \\
UB = &\inf_{z\in\mathcal{Z}} \left\{ P(z)E\left[Y_1|D=1,Z=z\right] + \left(1-P(z)\right) y^u_{z,1} \right\} \\
&- \sup_{z\in\mathcal{Z}} \left\{ \left(1-P(z)\right) E\left[Y_0|D=0,Z=z\right] + P(z) y^l_{z,0} \right\},
\end{aligned}
$$

where $P(z)$ denotes the propensity score $\Pr[D=1|Z=z]$, $y^l_{z,d}$ and $y^u_{z,d}$ represent the minimum and maximum values Y can take given $Z=z$ and $D=d$.

- Names for coefficient matrices are consistent with the ones in *LP_main*.

2.5 LP_Continuous

This file calculates bounds on the ATE with continuous Y and compares these results with Mogstad et al. (2018) for varying $\mathcal{Z}$. It generates Fig 9 in the paper.

- Support $\mathcal{Z}$ is controlled by the parameter *width_same*: if *width_same*=1, Z takes values from the sets $\{0,1\}$, $\{0,0.5,1\}$, $\{0,0.25,0.5,0.75,1\}$; if *width_same*=0, Z takes values from $\{0,1\}$, $\{0,1,2\}$, $\{0,1,2,3,4\}$.

- The counterfactual outcomes Y_0 and Y_1 are generated to be correlated directly with the latent variable U using the multivariate uniform variable function.

- Different from the discrete case, the polynomial orders are set to be 5 to reduce computational burden.

2.6 LP_CompareWithM_DiffWidth

This file continues the comparison between the current approach and Mogstad et al. (2018), similar to the file *LP_CompareWithMogstadEtAl*, but with different variations on $\mathcal{Z}$. It creates Fig 10 in the paper.

- The DGP models for D and Y remain consistent with the previous file. However, the values that Z can take are modified to include $\{0,1\}$, $\{0,1,2\}$, $\{0,1,2,3,4\}$ instead.

2.7 LP_CommonW

This file illustrates the results with common W and reverse IV W, highlighting that the identification power from W is due to its exogeneity rather than the reversibility.

- *common_W* indicates the type of W. *common_W*=1 indicates that W is the common exogenous variable, and *common_W*=0 indicates that W is the reverse IV.

- Names for coefficient matrices are consistent with the ones in *LP_main*.

2.8 LP_DiffK_Misspecification

This file presents the results when bernstein polynomials do not accurately fit the DGP, and quantifies the level of misspecification using the approximate Hausdorff distance to measure deviation between the partially identified bound and the true parameter.

- We applied a DGP with binary Z, D, Y. The MTR functions are generated with absolute Sine functions, which are well known to be challenging to approximate.

- If the true $m(u)$ is outside the identified bound, we consider the result as misspcification and calculate the distance between the true value and the bound. The distance between the true value of $m(u)$ and the closest point within the bounds is calculated across 100 grid points. The maximum deviation found across these points is used as the approximate Hausdorff distance, quantifying the misspecification.